Simple Performance Model for Ring and Line Cusp Ion Thrusters

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A model is developed for high magnetic field strength cusped thrusters, which results in a single algebraic equation describing the thruster performance. Such a simple result is made possible by formulating the model in terms of the average energy expended in producing ions in the discharge plasma and the fraction of these ions extracted into the beam. A key feature of the model is that it allows one to calculate the average plasma ion energy cost as a function of the propellant utilization, knowing only thruster geometric design parameters and the propellant gas. The model can be used to calculate the effects on performance resulting from changes in the extracted-ion fraction, the quality of the containment of primary electrons, the propellant mass flow rate, the propellant gas, and the transparency of the accelerator system to neutral atoms.

Nomenclature

A eq = equivalent amperes
A_g = grid area, m^2
C_0 = primary electron utilization factor, $= 4\sigma_0 \ell_e/(ev_0 A_g \phi_0)$, $(A\ eq)^{-1}$
e = electronic charge, 1.6×10^{-19} C
f_A = fraction of ion current produced that goes to anode potential surfaces
f_B = extracted-ion fraction
f_C = fraction of ion current produced that goes to cathode potential surfaces
J_A = ion current to anode potential surfaces, A
J_B = beam current, A
J_C = ion current to cathode potential surfaces, A
J_D = discharge current, A
J_E = cathode emission current, A
J_j = rate of production of jth excited state expressed as a current, A
J_L = current of primary electrons lost to the anode, A
J_M = current of Maxwellian electrons to the anode, A
J_P = ion production current, A
ℓ_e = primary electron containment length, m
$\dot{m}$ = propellant flow rate, A eq
n_e = plasma density, m^{-3}
n_0 = neutral density, m^{-3}
$\dot{n}_0$ = neutral loss rate, A eq
U_j = excitation energy of jth excited state, eV
U_+ = ionization energy, eV
V_C = potential (relative to cathode potential) from which electrons are accelerated to become primaries, V
V_D = discharge voltage, V
v_e = electron velocity, m/s
v_0 = neutral atom velocity, m/s
$\mathcal{V}_P$ = plasma volume, m^3
ϵ_B = average beam ion energy cost, eV
ϵ_M = average energy of Maxwellian electrons leaving the plasma at the anode, eV
ϵ_P = average plasma ion energy cost, eV
ϵ_P^* = baseline plasma ion energy cost, eV
ϵ_0 = average plasma ion energy cost considering ionization and excitation processes only, eV
η_u = propellant utilization efficiency
σ_0 = total inelastic collision cross section for primary electron-neutral atom collisions, m^2
σ_+ = ionization collision cross section, m^2
σ_j = excitation collision cross section of jth state, m^2
ϕ_0 = grid transparency to neutral atoms

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Introduction

ION thruster performance is typically measured in terms of the power (in watts) required to produce, but not accelerate, an ion beam current of 1 A at a given propellant utilization efficiency. This power, called the specific discharge power (watts/beam ampere), must be supplied to effect thruster operation, but it does not produce any thrust by itself. Thus, it is desirable to minimize the specific discharge power while maintaining a high propellant utilization. The variation in specific discharge power with the propellant utilization is known as a performance curve.

In order to improve thruster performance, i.e., reduce the specific discharge power, it is desirable to have a theoretical model that describes the effects of thruster design variables and operating parameters on the performance curve. Theoretical models have been developed in the past to predict the performance curves of both low magnetic field strength (Kaufman-type) thrusters[1] and high field strength cusped thrusters.[2] However, these models typically consist of a complex set of equations that must be solved iteratively by a computer. This complexity makes it difficult to identify the dominant mechanisms controlling thruster performance. Earlier work on Kaufman-type thrusters by Masek[3] and Knauer[4] described thruster performance physically in terms of the average energy expended to produce ions in the discharge chamber plasma. This work resulted in an increased understanding of some of the phenomena affecting thruster performance but stopped short of providing a complete model that could be used to predict the performance curve.

In this paper, a model that describes the performance of high flux density, cusped magnetic field thrusters is developed. The essence of the model is a single algebraic equation formulated in terms of the average energy expended in producing an ion in the discharge chamber plasma (the plasma ion energy cost) and the fraction of the ions produced that are extracted into the beam. A key feature of the model is that it allows one to calculate the average plasma ion energy cost as a function of the propellant mass flow rate and propellant utilization, knowing only thruster geometric design parameters and the propellant gas. From this, the beam ion energy cost, which is numerically equal to the specific discharge power, is calculated.

The model can be used to calculate the changes in thruster performance due to variations in such parameters as propellant flow rate, propellant gas, discharge voltage, extracted-ion fraction, accelerator grid transparency to neutral atoms, and containment of primary electrons. Improvements in thruster design suggested by the model are discussed. Experiments testing the validity of several features of the model are described in two companion articles.[5,6]

Theoretical Development

Beam Ion Energy Cost

The specific discharge power, or energy cost per beam ion, (ϵ_B) is defined by the equation,

$$\epsilon_B = (J_D - J_B)V_D/J_B \quad \text{(eV/beam ion)} \tag{1}$$

The beam current J_B in Eq. (1) is subtracted from the discharge current so that the energy that goes into accelerating the beam ions through the discharge voltage V_D is not charged to the energy cost per beam ion.

In a similar manner, the average energy expended in creating ions in the discharge chamber plasma may be defined as

$$\epsilon_P = [J_D - (J_C + J_B)]V_D/J_P \quad \text{(eV/plasma ion)} \tag{2}$$

By analogy to Eq. (1), the $(J_C + J_B)$ term is subtracted from the discharge current so that the energy that goes into accelerating these ions out of the discharge chamber plasma into the walls or the beam is not included in the ion production cost. Rearranging Eq. (2) yields

$$\epsilon_P = \frac{(J_D - J_B)}{J_B} V_D \left(\frac{J_B}{J_P}\right) - \frac{J_C}{J_P} V_D \tag{3}$$

Defining the fractions of ion current produced that go into the beam and to cathode potential surfaces as

$$f_B = J_B/J_P, \qquad f_C = J_C/J_P \tag{4}$$

respectively,‡ and using Eq. (1) in Eq. (3) yield

$$\epsilon_P = \epsilon_B f_B - f_C V_D \tag{5}$$

Solving this equation for ϵ_B gives the result

$$\epsilon_B = \epsilon_P/f_B + f_C V_D/f_B \tag{6}$$

This equation describes the beam ion energy cost as a function of the plasma ion energy cost ϵ_P, the extracted-ion fraction f_B, the fraction of ion current to cathode potential surfaces f_C, and the discharge voltage V_D.

The first term on the right-hand side of Eq. (6) represents the energy loss associated with producing ions in the discharge chamber and extracting only a fraction of them into the beam. Ions that are not extracted into the beam go to the walls of the discharge chamber, where they recombine. The resulting atoms must then be reionized before they can contribute to the beam current. Thus, the factor $1/f_B$ may be interpreted as the average number of times a beam ion undergoes ionization before being extracted into the beam.

The second term on the right-hand side of Eq. (6) represents the energy wasted in accelerating plasma ions into interior cathode potential surfaces. This process results in both a discharge energy loss and in the sputter erosion of these surfaces. In order to generate performance curves using Eq. (6), one must be able to specify the behavior of each of the terms on the right-hand side of Eq. (6) as a function of the propellant utilization.

‡The relationship between these fractions may be found by dividing the equation $J_P = J_B + J_C + J_A$ by the total ion production current J_P to get $f_B + f_C + f_A = 1$, where $f_A = J_A/J_P$.

Plasma Ion Energy Cost

The plasma ion energy cost parameter ϵ_P, appearing in Eq. (6) and defined by Eq. (2), reflects all mechanisms of energy loss from the discharge chamber plasma except for the acceleration of ions out of the plasma through the discharge voltage. Specifically, ϵ_P includes energy losses associated with the following phenomena: direct primary electron loss to the anode, Maxwellian electron collection by the anode, excitation of neutral atoms, excitation of ionic states (which will be neglected), and hollow cathode operation. To derive an expression for the plasma ion energy cost as a function of the propellant utilization, a power balance is made on the discharge chamber plasma represented in Fig. 1. The primary electrons are assumed to be accelerated from a potential that is V_C volts above cathode potential to the potential of the bulk plasma, which is assumed to be near anode potential, i.e., at the discharge voltage V_D above cathode potential. In addition, if it is assumed that only the discharge power supply is used to sustain the discharge, then the rate at which energy is supplied to the plasma by the primary electrons is given by $J_E(V_D - V_C)$. The "missing" power $J_E V_C$ is used to operate the hollow cathode. Energy is lost from the plasma primarily by the flux of four types of energy carriers across the plasma boundaries: ions, photons (emitted by de-excitation of excited propellant atoms), Maxwellian electrons, and primary electrons. The ions and photons are lost to all interior thruster surfaces, whereas the Maxwellian and primary electrons are assumed to be lost to the anode surfaces only. In steady state the rate at which energy is supplied to the plasma must be equal to the rate at which it is lost; thus,

$$J_E(V_D - V_C) = J_P U_+ + \sum_j J_j U_j + J_M \epsilon_M + J_L(V_D - V_C) \tag{7}$$

Dividing Eq. (7) by the ion production current J_P and recognizing that the emission current J_E is related to the discharge current by

$$J_E = J_D - (J_C + J_B) \tag{8}$$

allow Eq. (7) to be written as

$$\epsilon_P = U_+ + \sum_j \frac{J_j U_j}{J_P} + \frac{J_M \epsilon_M}{J_P} + \frac{J_L V_D}{J_P} + \frac{J_E V_C}{J_P} - \frac{J_L V_C}{J_P} \tag{9}$$

where Eq. (2) has been used. The rate at which the jth excited state is produced is given by

$$J_j = e n_0 n_e \langle \sigma_j v_e \rangle \mathcal{V}_P \tag{10}$$

where $\langle \sigma_j v_e \rangle$ represents the product of the jth excitation collision cross section and the electron velocity averaged over the electron speed distribution. Similarly, the ion production cur-

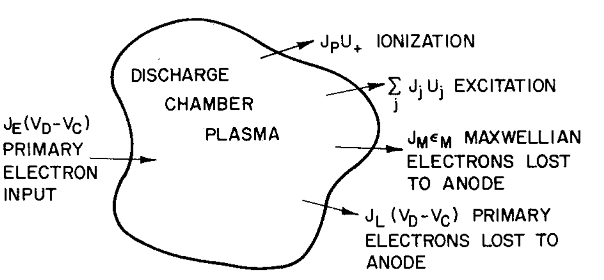

Fig. 1 Discharge power balance schematic.

rent is given by

$$J_P = en_0 n_e \langle \sigma_+ v_e \rangle V_P \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (9) yields

$$\epsilon_P = \epsilon_0 + \frac{J_M \epsilon_M}{J_P} + \frac{J_L V_D}{J_P} + \frac{J_E V_C}{J_P} - \frac{J_L V_C}{J_P} \tag{12}$$

where the parameter ϵ_0 is defined by

$$\epsilon_0 \equiv U_+ + \sum_j \frac{\langle \sigma_j v_e \rangle U_j}{\langle \sigma_+ v_e \rangle} \tag{13}$$

This parameter accounts for the energy expended in ionization and excitation reactions and may be calculated in the manner described by Dugan and Sovie.[8]

The third term on the right-hand side of Eq. (12) may be written as

$$\frac{J_L V_D}{J_P} = \frac{J_L}{J_E}\left[\frac{J_E V_D}{J_P}\right] = \frac{J_L}{J_E}\epsilon_P \tag{14}$$

where the last step was made using Eqs. (2) and (8). The ratio J_L/J_E is simply the fraction of primary electrons emitted by the cathode, which are collected by the anode before having any inelastic collisions. This fraction is given by the survival equation[9]

$$J_L/J_E = \exp[-\sigma_0 n_0 \ell_e] \tag{15}$$

where ℓ_e is the average distance a primary electron would travel in the discharge chamber before being collected by the anode—assuming it had no inelastic collisions. Combining Eqs. (12), (14), and (15) yields

$$\epsilon_P = \epsilon_0 + \frac{J_M \epsilon_M}{J_P} + \epsilon_P \exp[-\sigma_0 n_0 \ell_e] + \epsilon_P \frac{V_C}{V_D} - \epsilon_P \frac{V_C}{V_D}\exp[-\sigma_0 n_0 \ell_e] \tag{16}$$

The current of Maxwellian electrons to the anode is the sum of the secondary electrons liberated in the ionization process and the thermalized primary electrons; thus,

$$J_M = J_P + (J_E - J_L) = J_P + J_E(1 - J_L/J_E) \tag{17}$$

Using Eqs. (15) and (17) in Eq. (16) yields

$$\epsilon_P = \epsilon_0 + \{J_P + J_E(1 - \exp[-\sigma_0 n_0 \ell_e])\}\frac{\epsilon_M}{J_P} + \epsilon_P\left\{\exp[-\sigma_0 n_0 \ell_e] + (1 - \exp[-\sigma_0 n_0 \ell_e])\frac{V_C}{V_D}\right\} \tag{18}$$

Finally, using Eqs. (2) and (8) and solving Eq. (18) for ϵ_P results in

$$\epsilon_P = \left[\frac{\epsilon_0 + \epsilon_M}{1 - (V_C + \epsilon_M)/V_D}\right]\{1 - \exp[-\sigma_0 n_0 \ell_e]\}^{-1} \tag{19}$$

The factor $1 - \exp[-\sigma_0 n_0 \ell_e]$ may be interpreted as the probability that a primary electron will have an inelastic collision before it is collected by the anode. This is analogous to the fast neutron nonleakage probability used in nuclear reactor physics.[10] The neutral density n_0 may be expressed in terms of the propellant flow rate ($\dot{m}$) and propellant utilization (η_u) by equating the rate at which propellant enters and leaves the discharge chamber, i.e.,

$$\dot{m} = J_B + \dot{n}_0 \tag{20}$$

where $\dot{m}$ and $\dot{n}_0$ are in units of equivalent amperes (A eq). The neutral flow rate from the thruster may be calculated from the kinetic theory of gases as

$$\dot{n}_0 = \tfrac{1}{4} n_0 e v_0 A_g \phi_0 \tag{21}$$

where ϕ_0 is the effective transparency of the grids to neutral atoms. Combining Eqs. (20) and (21) yields

$$n_0 = [4\dot{m}(1 - \eta_u)]/e v_0 A_g \phi_0 \tag{22}$$

where $\eta_u = J_B/\dot{m}$ was used. Thus, Eq. (19) may be written

$$\epsilon_P = \epsilon_P^*\{1 - \exp[-C_0 \dot{m}(1 - \eta_u)]\}^{-1} \tag{23}$$

where

$$C_0 = 4\sigma_0 \ell_e / e v_0 A_g \phi_0 \tag{24}$$

and

$$\epsilon_P^* = \frac{\epsilon_0 + \epsilon_M}{1 - [(V_C + \epsilon_M)/V_D]} \tag{25}$$

Equation (23) provides a very simple method for calculating the plasma ion energy cost as a function of the propellant utilization. Experimental results[5,6] indicate that under many conditions the parameters C_0 and ϵ_P^* may be taken to be independent of the propellant utilization. Substitution of Eq. (23) into Eq. (6) yields the following simple equation describing the performance curve for a given thruster design

$$\epsilon_B = \frac{\epsilon_P^*}{f_B\{1 - \exp[-C_0 \dot{m}(1 - \eta_u)]\}} + \frac{f_C}{f_B}V_D \tag{26}$$

For design purposes the parameters f_B and f_C, in addition to C_0 and ϵ_P^*, may be taken to be independent of the propellant utilization and flow rate. These parameters do, however, depend strongly on the thruster design. Indeed, these four parameters determine the performance of a given thruster design.

The quantity C_0 describes the degree to which primary electrons interact with neutral atoms. Thus, it is referred to as the primary electron utilization factor. This factor depends on the quality of the primary electron containment (through ℓ_e), the quality of the containment of neutrals (through A_g, ϕ_0, and v_0), and the propellant gas (through σ_0 and v_0). Recall that the primary electron containment length ℓ_e may be interpreted as the average distance a primary electron would travel in the discharge chamber before being lost to the anode—assuming it had no inelastic collisions. Magnetic fields in all discharge chamber designs serve the function of increasing this length. Although an effective means of determining ℓ_e remains to be developed, it is believed that this parameter is a function primarily of the thruster geometry, magnetic field configuration, and cathode location.

Equation (23) suggests that through the factor C_0 the plasma ion cost should depend on:

1) The propellant, which determines the total inelastic collision cross section σ_0, and atomic mass, which affects the thermal neutral velocity v_0.
2) The wall temperature, which affects the neutral velocity.
3) The transparency of the grids to neutrals, ϕ_0.
4) The area through which the beam is extracted, A_g.
5) The discharge voltage, which determines the primary electron energy and thus affects the value of the cross section σ_0.

The baseline plasma ion energy cost ϵ_P^* defined by Eq. (25) depends on a number of energy loss mechanisms including 1) the relative amount of energy expended in excitations compared to ionization of neutral atoms through ϵ_0, 2) the average energy of the Maxwellian electrons that leave the plasma ϵ_M, and 3) the efficiency with which the hollow cathode operates. The cathode efficiency is reflected in the value of V_C, which represents plasma potential from which the electrons are supplied. Inefficient cathode operation results in high values of V_C and corresponding poor overall thruster performance. For thermionic cathodes $V_C = 0$, however, additional heater power must be supplied to effect its operation. For thrusters equipped with a cathode pole piece/baffle assembly, V_C should be taken as the plasma potential in the cathode discharge region, i.e., the region between the cathode and baffle. Thus, the power represented by $J_E V_C$ in this case goes into both the operation of the hollow cathode and the operation of the cathode region discharge. The resulting high values of V_C in this case would be expected to produce poorer overall thruster performance. Elimination of the separate cathode discharge region should, therefore, improve the performance.

In general, the parameter ϵ_0, defined by Eq. (13), depends on the electron energy distribution. In spite of the complexity associated with the computation of the parameter ϵ_P^*, experiments[5,6] indicate that for design purposes it may be taken to be constant for a given discharge chamber magnetic field/anode configuration, propellant type, and discharge voltage. A simple method for the calculation of ϵ_P^* has been developed, which yields calculated values that agree well with measured ones.[7]

Although the model cannot yet be used to predict the performance of completely new thruster designs, it provides a clear physical picture of the phenomena affecting the performance. Equation (23) describes the plasma ion energy cost in terms of the loss of primary electrons to the anode. At high values of the neutral density parameter, the neutral density in the discharge chamber is large, and the probability is high that all of the primary electrons will undergo inelastic collisions with neutral atoms and that none will be lost directly to the anode. In this case, the discharge chamber will be producing ions for the minimum or baseline energy cost ϵ_P^*.

As the beam current is increased (at a constant propellant flow rate), the propellant utilization increases, causing the neutral density parameter, and thus the neutral density, to decrease [see Eqs. (20-22)]. The decrease in neutral density increases the likelihood of a primary electron reaching the anode without having an inelastic collision. This direct loss of primary electron energy increases the overall plasma ion energy cost according to Eq. (23) and consequently increases the beam ion energy cost according to Eq. (6). The shape of the performance curve is largely determined by the direct loss of primary electrons. Consequently, any design change that decreases the likelihood of the direct loss of primary electrons (without decreasing the extracted-ion fraction) should improve the thruster's performance. The probability of direct primary electron loss is determined through the parameter C_0. This parameter may be increased by either increasing ℓ_e, which makes it harder for primary electrons to escape the plasma, or by making it harder for neutral atoms to escape the discharge chamber.

Since the plasma ion energy cost depends on the neutral density parameter and, hence, on the neutral density itself, the model provides an explanation for the difference in discharge chamber performance observed for thruster operation with and without ion beam extraction. It has been observed that the performance (eV/beam ion) extrapolated from thruster operation without beam extraction is generally significantly better than the performance measured with beam extraction.[11,12] This may be understood by noting that thruster operation without beam extraction should be characterized by higher discharge chamber neutral densities than operation with beam extraction at the same flow rate. Further, Eq. (23) written as

$$\epsilon_P = \epsilon_P^* \{1 - \exp[-\sigma_0 n_0 \ell_e]\}^{-1} \tag{27}$$

indicates that the plasma ion energy cost should be higher for smaller values of n_0. Thus, the performance with beam extraction should be poorer than that extrapolated from data obtained without beam extraction. Finally, it is noted that careful application of the performance model developed here should allow more meaningful extrapolation of data taken without beam extraction to operation with beam extraction.

Thruster Design Impact

A simple algebraic equation for the performance curve for cusped magnetic field thrusters is given by Eq. (26). This equation, which is believed to be applicable to either ring or line cusp configurations, suggests that thruster performance is a function of only four configuration/propellant-dependent parameters, ϵ_P^*, C_0, f_B, and f_C, and two operating parameters, $\dot{m}$ and V_D. The effect of each of these parameters on performance may be easily investigated analytically through the use of Eq. (26). This is done by specifying values for each of the parameters in the equation and then varying each parameter individually to determine its effect on the performance.

An example of this procedure is shown in Fig. 2, where the effect of the extracted-ion fraction f_B on the performance is demonstrated. The values of the parameters $\dot{m}$, C_0, f_C, V_D, and ϵ_P^*, which were used in Eq. (26) to generate the curves in this figure, are specified in the legend. This figure indicates the strong effect of the extracted-ion fraction on the performance. Changes in f_B cause the performance curve to shift up or down but do not substantially change its shape. Figure 2 also indicates that for a value of $\epsilon_P^* = 50$ eV, the extracted-ion fraction must be greater than ~ 0.6 for the beam ion energy cost to be under 100 eV/beam ion. Clearly, it is desirable to have f_B as large (near unity) as possible.

The effect of the primary electron utilization factor C_0 on performance is given in Fig. 3. This parameter also has a strong effect on the performance. Indeed, it is this parameter that primarily determines the shape of the performance curve, with larger values of C_0 corresponding to curves with more sharply defined "knees." The definition of the primary electron utilization factor, Eq. (24), suggests a number of ways in which the value of C_0 may be increased, for example, by using a propellant gas characterized by a larger in-

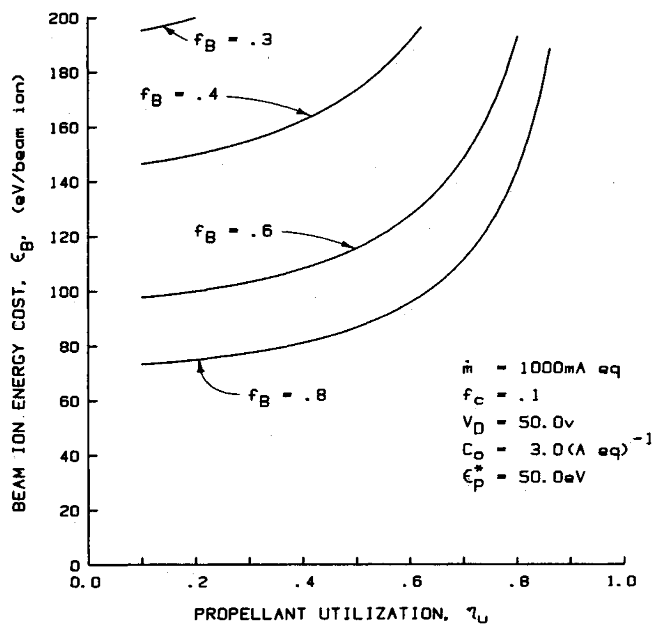

Fig. 2 Effect of extracted-ion fraction on performance.

elastic collision cross section σ_0 and a larger atomic mass (resulting in a smaller neutral velocity v_0). One may also increase this factor by decreasing the effective grid transparency to neutral atoms ϕ_0; this must be done, of course, without increasing the accelerator grid impingement current. For thruster designs with nonuniform beam profiles, tailoring the accelerator grid hole size to match the radial current density profile might be a useful way to minimize ϕ_0. Also, three grid systems might be expected to have smaller values of ϕ_0 than two grid systems.

Most importantly, C_0 may be increased by increasing the primary electron containment length ℓ_e. This length corresponds to the average distance a primary electron would travel in the discharge chamber before being collected by an anode surface, provided it had no inelastic collisions. As mentioned previously, the primary function of the magnetic field in all thruster designs is to increase this length. In cusped field thrusters, primary electrons are lost to the anode through the cusps. Thus, ℓ_e may be increased by decreasing the number of cusps at anode potential or increasing the magnetic flux density at the cusps, but only up to a point. Reductions in effective anode cusp area below a certain limit will result in unstable operation of the discharge.[13]

Equation (24) also suggests that the primary electron utilization factor may be increased by masking down the area of the grids through which the beam is extracted, A_g. However, decreasing A_g in this manner will also lead to a large reduction in the extracted-ion fraction and, consequently, an overall reduction in performance.

Increases in propellant flow rate, as shown in Fig. 4, should improve discharge chamber performance. The maximum flow rate at which the thruster can be operated is limited by the ability of the accelerator system to extract the ion current directed toward it. The effect of flow rate on performance is less dramatic for thruster designs characterized by larger values of the primary electron utilization factor C_0, as shown in Fig. 5. This suggests that ion thrusters characterized by large values of C_0 may be effectively throttled with little penalty in overall thruster efficiency.

The discharge chamber model also suggests performance changes that might be expected from scaling thruster designs to different discharge chamber diameters. This may be seen by examining the product of the primary electron utilization factor and the propellant mass flow rate. This product is a dimensionless quantity, namely,

$$C_0\dot{m} = 4\sigma_0\ell_e\dot{m}/ev_0\phi_0A_g \tag{28}$$

For constant beam current densities, thruster scaling should be accomplished such that the ratio of mass flow rate to active grid area $\dot{m}/A_g$ is constant. Thus, for the same propellant and grid transparency to neutrals, Eq. (28) suggests that the discharge chamber performance depends only on the primary electron containment length ℓ_e. It is believed that this length should increase with thruster diameter, and this suggests that larger-diameter thrusters should be more efficient than smaller ones. But this is not the whole story. The effect of thruster diameter on the extracted-ion fraction must also be considered. The above conclusion remains true, however, for thruster scaling accomplished in such a way that f_B does not decrease.

Finally, Eq. (26) indicates that the changes in the baseline plasma ion energy cost ϵ_P^* should merely shift the performance curves up or down. The amount of the shift increases for smaller values of f_B. For thrusters that utilize hollow cathodes, the efficiency of the cathode operation—characterized by V_C in Eq. (25)—has a strong effect on the value of ϵ_P^*, especially at low discharge voltages.[7]

Neutral Atom Loss Considerations

Rewriting Eq. (20), one obtains an expression for the neutral atom loss rate from the discharge chamber

$$\dot{n}_0 = \dot{m} - J_B = \dot{m}(1-\eta_u) \tag{29}$$

Substituting this into Eq. (26) and solving for the neutral loss rate yield

$$\dot{n}_0 = -\frac{1}{C_0}\ell n\left[1 - \frac{\epsilon_P^*}{f_B\epsilon_B - f_C V_D}\right] \tag{30}$$

This equation gives the value of the neutral loss rate as a function of the beam ion energy cost. For a specified thruster geometry and discharge voltage, the design parameters C_0, ϵ_P^*, f_B, f_C, and V_D may be taken to be approximately constant. Thus, since the propellant mass flow rate does not appear on the right-hand side of Eq. (30), this equation predicts that the neutral loss rate $\dot{n}_0$ is independent

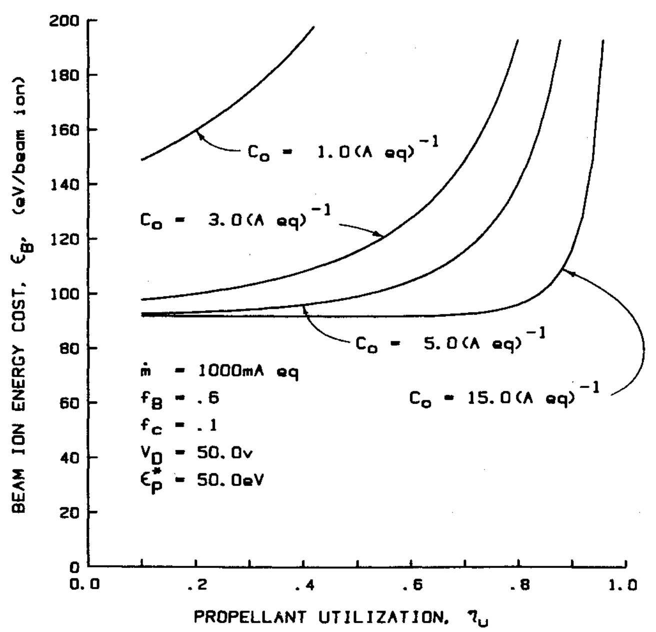

Fig. 3 Effect of primary electron utilization on performance.

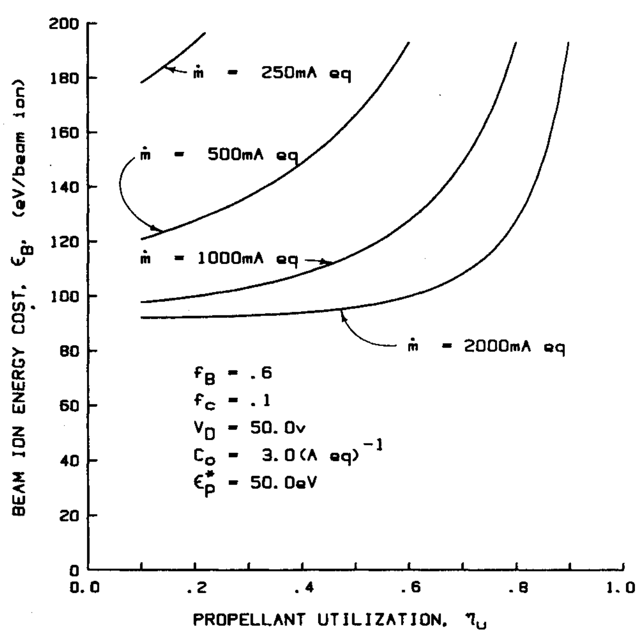

Fig. 4 Effect of flow rate on performance for $C_0 = 3$ (A eq)$^{-1}$.

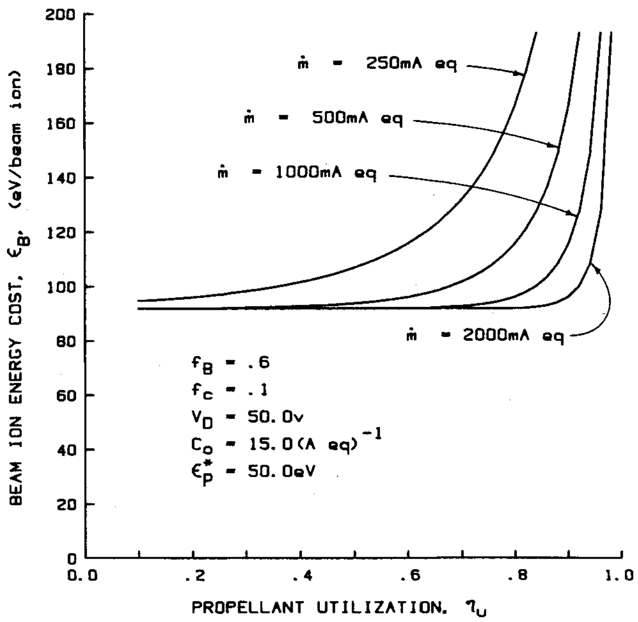

Fig. 5 Effect of flow rate on performance for $C_0 = 15$ (A eq)$^{-1}$.

of the flow rate at a constant value of the energy cost per beam ion ϵ_B. This same conclusion was reached originally by Kaufman and Cohen[14] in their constant neutral loss rate theory developed for low magnetic field strength discharge chamber designs.

Conclusions

A very simple thruster performance model has been developed which is easy to apply as an aid in the design of new thrusters. This model describes thruster performance (beam ion energy cost vs propellant utilization) in terms of only four configuration/propellant-dependent parameters, namely: the primary electron utilization factor C_0, the baseline plasma ion energy cost ϵ_P^*, the extracted-ion fraction f_B, and the cathode potential surface ion fraction f_C; and two operating parameters: the propellant flow rate $\dot{m}$ and the discharge voltage V_D. The model suggests that the direct loss of primary electrons to the anode is the principal factor causing the energy cost of a beam ion to rise at high propellant utilization levels. Indeed, to a large extent, the shape of the thruster performance curve is determined by this direct loss of primary electrons.

The model suggests that improved thruster performance should be characterized by large extracted-ion fractions, long primary electron containment lengths, small effective grid transparencies to neutral atoms, and operation at high propellant flow rates. Thruster designs characterized by large values of C_0 should allow efficient throttling.

Future work should focus on determining the dependence of f_B and ℓ_e on the thruster geometry and magnetic field configuration.

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